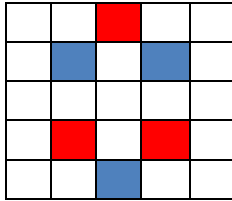


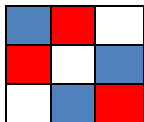
Arrow puzzles – hexagon rule

Two types of configuration seem useful (due to most arrows contributing equally to the totals because they pass through both a red and a blue vertex):



For this configuration, the difference between the red and blue vertex totals corresponds to the difference between:

- The count of horizontal arrows going through row 1 and twice the count through row 4
- The count of horizontal arrows going through row 5 and twice the count through row 2



The differences work in a similar way for this configuration. Below is an example, which works through part of GP 2022 Round 8 puzzle 9. The letters denote lines passing through the boxes as shown – none will actually pass through G or N but keeping them helps avoid typos.

	A	B	C	D	E	F	G
	2	2	4	4	3	2	H
	2	2	3	2	1	4	I
	5	6	3	3	6	4	J
	3	2	2	3	2	2	K
	3	2	5	4	2	3	L
	3	6	4	4	5	3	M
		S	R	Q	P	O	N

Applying the second configuration to the top-left 3x3 box:

	B	C	D	E	F
	2	2	4		
	2	2	3		
	5	6	3		

The red vertices total 11, which is 4 more than the blue vertices, which total 7. The “uncancelled directions” are the diagonal lines shown. Diagonals C and E pass through two vertices, so need to be counted twice. This gives

$B + 2 \cdot E - (F + 2 \cdot C) = 4$. Rearranging this, and following a similar process moving to the right and down, gives

$$(B-F)+2*(E-C) = 4$$

$$(C-G)+2*(F-D) = -2$$

$$(D-H)+2*(G-E) = -5 \quad \text{Immediately gives } E=2 \text{ (i.e double arrow)}$$

$$(E-I)+2*(H-F) = 5 \quad \text{Immediately gives } H=2, F=0$$

$$(F-J)+2*(I-G) = 2$$

$$(G-K)+2*(J-H) = -5 \quad \text{Immediately gives } H=2, J=0$$

$$(H-L)+2*(K-I) = 2$$

This seems to me the best place to stop and draw some arrows in to see if there are any implications that follow on from this (e.g. $F=0$ implies that the right-hand top box must point down, and the bottom left-hand box must point right). But, it's possible to take the above brute-force approach further:

Substituting for E, F, H, J (together with $G=0$) gives

$$B = 2C \quad \text{This implies } C \text{ is } 0 \text{ or } 1$$

$$C = 2D-2 \quad \text{This implies } C \text{ is } 0 \text{ or } 2; \text{ hence } C=B=0 \text{ and } D=1$$

$$D=H-1 \quad \text{So } H=2$$

$$E=I+1 \quad E=2 \text{ (see following line)}$$

$$I=1$$

$$K=1$$

$$2*(K-I) = L \quad \text{So } L=0, \text{ and we have complete information for whether or not any arrows are along the / line (but not which direction they point if there is only one)}$$

This seems more useful in some situations than others – the above problem was a low-point one and so other puzzles might not fall so easily. But, the following principles are probably the most important:

- If the totals differ by 5 or 6 then you can put a double line in
- Whether the differences between the totals is even or odd can give useful information about how the “outer vertices” (the top-left and bottom right in the above worked example) might differ (particularly if one of those vertices is on a diagonal)
- Hexagons (in the second configuration) which overlap the main diagonal can lead to quick progress